

Time : 3 hrs.

Answers & Solutions
for
JEE (MAIN)-2018
(Physics, Mathematics and Chemistry)

M.M. : 360

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **90** questions. The maximum marks are **360**.
3. There are **three** parts in the question paper A, B, C consisting of **Physics, Mathematics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted 4 (**four**) marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
5. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction No. 4 above.
6. For writing particulars/markings responses on **Side-1** and **Side-2** of the Answer Sheet use **only Black Ball Point Pen** provided in the examination hall.
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.

PART-A : PHYSICS

1. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c . The values of p_d and p_c are respectively

- (1) (0, 0) (2) (0, 1)
(3) (.89, .28) (4) (.28, .89)

Answer (3)

Sol. $mu = mv_1 + 2m \times v_2$... (i)

$u = (v_2 - v_1)$... (ii)

$\Rightarrow v_1 = -\frac{u}{3}$

$$\therefore \frac{\Delta E}{E} = p_d = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{3}\right)^2}{\frac{1}{2}mu^2}$$

$$= \frac{8}{9} = 0.89$$

And $mu = mv_1 + (12m) \times v_2$... (iii)

$u = (v_2 - v_1)$... (iv)

$\Rightarrow v_1 = -\frac{11}{13}u$

$$\therefore \frac{\Delta E}{E} = p_c = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{11}{13}u\right)^2}{\frac{1}{2}mu^2} = \frac{48}{169} = 0.28$$

2. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly

- (1) $2.35 \times 10^2 \text{ N/m}^2$ (2) $4.70 \times 10^2 \text{ N/m}^2$
(3) $2.35 \times 10^3 \text{ N/m}^2$ (4) $4.70 \times 10^3 \text{ N/m}^2$

Answer (3)

Sol. $F = nmv \cos \theta \times 2$

$P = \frac{F}{A} = \frac{2nmv \cos \theta}{A}$

$$= \frac{2 \times 10^{23} \times 3.32 \times 10^{-27} \times 10^3}{\sqrt{2} \times 2 \times 10^{-4}} \text{ N/m}^2$$

$= 2.35 \times 10^3 \text{ N/m}^2$

3. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the

radius of the sphere, $\left(\frac{dr}{r}\right)$, is

- (1) $\frac{mg}{3Ka}$ (2) $\frac{mg}{Ka}$
(3) $\frac{Ka}{mg}$ (4) $\frac{Ka}{3mg}$

Answer (1)

Sol. $K = -V \frac{dP}{dV}$

$\Rightarrow \frac{-dV}{V} = \frac{dP}{K} = \frac{mg}{Ka}$

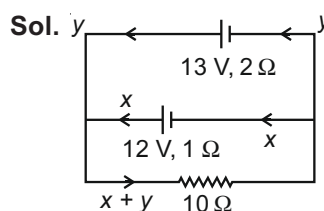
$\Rightarrow \frac{-3dr}{r} = \frac{mg}{Ka}$

$\Rightarrow \frac{dr}{r} = -\frac{mg}{3Ka}$

4. Two batteries with e.m.f 12 V and 13 V are connected in parallel across a load resistor of 10Ω . The internal resistances of the two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between

- (1) 11.4 V and 11.5 V
(2) 11.7 V and 11.8 V
(3) 11.6 V and 11.7 V
(4) 11.5 V and 11.6 V

Answer (4)



Applying KVL in loops

$$12 - x - 10(x + y) = 0$$

$$\Rightarrow 12 = 11x + 10y \quad \dots(i)$$

$$13 = 10x + 12y \quad \dots(ii)$$

$$\text{Solving } x = \frac{7}{16} \text{ A, } y = \frac{23}{32} \text{ A}$$

$$V = 10(x + y) = 11.56 \text{ V}$$

$$\text{Aliter : } r_{eq} = \frac{2}{3} \Omega, R = 10 \Omega$$

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} \Rightarrow E_{eq} = \frac{37}{3} \text{ V}$$

$$V = \frac{E_{eq}}{R + r_{eq}} R = 11.56 \text{ V}$$

5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$.

Its total energy is

- (1) Zero
 (2) $-\frac{3}{2} \frac{k}{a^2}$
 (3) $-\frac{k}{4a^2}$
 (4) $\frac{k}{2a^2}$

Answer (1)

$$\text{Sol. } F = \frac{-dU}{dr} \quad \left[U = -\frac{k}{2r^2} \right]$$

$$\frac{mv^2}{r} = \frac{k}{r^3} \quad [\text{This force provides necessary centripetal force}]$$

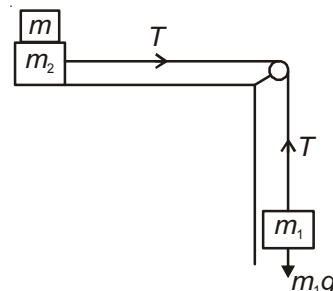
$$\Rightarrow mv^2 = \frac{k}{r^2}$$

$$\Rightarrow K.E = \frac{k}{2r^2}$$

$$\Rightarrow P.E = -\frac{k}{2r^2}$$

Total energy = Zero

6. Two masses $m_1 = 5 \text{ kg}$ and $m_2 = 10 \text{ kg}$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is



- (1) 43.3 kg
 (2) 10.3 kg
 (3) 18.3 kg
 (4) 27.3 kg

Answer (4)

Sol. To stop the moving block m_2 , acceleration of m_2 should be opposite to velocity of m_2

$$m_1 g < \mu(m + m_2)g$$

$$\Rightarrow 5 < 0.15(10 + m_2)$$

$$\Rightarrow m_2 > 23.33 \text{ kg}$$

$\therefore$ Minimum mass = 27.3 kg (according to given options)

7. If the series limit frequency of the Lyman series is ν_L , then the series limit frequency of the Pfund series is

- (1) $\nu_L/16$
 (2) $\nu_L/25$
 (3) $25 \nu_L$
 (4) $16 \nu_L$

Answer (2)

$$\text{Sol. } h\nu_L = E \left[\frac{1}{12} - \frac{1}{\infty} \right] = E$$

$$h\nu_P = E \left[\frac{1}{5^2} - \frac{1}{\infty} \right] = \frac{E}{25}$$

$$\Rightarrow \nu_P = \frac{\nu_L}{25}$$

8. Unpolarized light of intensity I passes through an ideal polarizer A . Another identical polarizer B is placed behind A . The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B . The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is

- (1) 45° (2) 60°
(3) 0° (4) 30°

Answer (1)

Sol. Polaroids A and B are oriented with parallel pass axis

Let polaroid C is at angle θ with A then it makes θ with B also.

$$\therefore \frac{I}{8} = \left(\frac{I}{2} \times \cos^2 \theta \right) \times \cos^2 \theta$$

$$\Rightarrow \cos^2 \theta = \frac{1}{2}$$

$$\Rightarrow \theta = 45^\circ$$

9. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n , (A, B are constants)

- (1) $\Lambda_n^2 \approx A + B\lambda_n^2$
(2) $\Lambda_n^2 \approx \lambda$
(3) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$
(4) $\Lambda_n \approx A + B\lambda_n$

Answer (3)

Sol. $P_n = \frac{h}{\lambda_n}, P_g = \frac{h}{\lambda_g}$

$$k = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}, E = -k = -\frac{h^2}{2m\lambda^2}$$

$$E_n = -\frac{h^2}{2m\lambda_n^2}, E_g = -\frac{h^2}{2m\lambda_g^2}$$

$$E_n - E_g = \frac{h^2}{2m} \left(\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2} \right) = \frac{hc}{\Lambda_n}$$

$$\frac{h^2}{2m} \left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2 \lambda_n^2} \right) = \frac{hc}{\Lambda_n}$$

$$\Lambda_n = \frac{2mc}{h} \left(\frac{\lambda_g^2 \lambda_n^2}{\lambda_n^2 - \lambda_g^2} \right)$$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h} \frac{\lambda_n^2}{\lambda_n^2 \left(1 - \frac{\lambda_g^2}{\lambda_n^2} \right)}$$

$$= \frac{2mc\lambda_g^2}{h} \left[1 - \frac{\lambda_g^2}{\lambda_n^2} \right]^{-1}$$

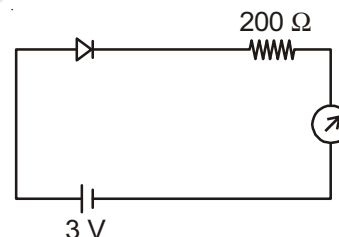
$$= \frac{2mc\lambda_g^2}{h} \left[1 + \frac{\lambda_g^2}{\lambda_n^2} \right]$$

$$= \frac{2mc\lambda_g^2}{h} + \left(\frac{2mc\lambda_g^4}{h} \right) \frac{1}{\lambda_n^2}$$

$$= A + \frac{B}{\lambda_n^2}$$

$$A = \frac{2mc\lambda_g^2}{h}, B = \frac{2mc\lambda_g^4}{h}$$

10. The reading of the ammeter for a silicon diode in the given circuit is



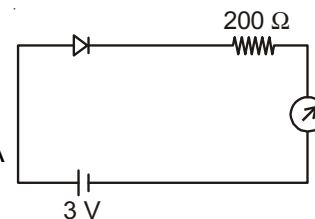
- (1) 11.5 mA
(2) 13.5 mA
(3) 0
(4) 15 mA

Answer (1)

Sol. $I = \frac{V - V_{\text{diode}}}{R}$

$$= \left[\frac{3 - 0.7}{200} \times 1000 \right] \text{ mA}$$

$$= 11.5 \text{ mA}$$



11. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B . The relation between r_e, r_p, r_α is

- (1) $r_e < r_p < r_\alpha$ (2) $r_e < r_\alpha < r_p$
(3) $r_e > r_p = r_\alpha$ (4) $r_e < r_p = r_\alpha$

Answer (4)

Sol. $r = \frac{\sqrt{2mk}}{qB}$

$$\frac{r_\alpha}{r_p} = \frac{\sqrt{2m_\alpha}}{q_\alpha} \times \frac{q_p}{\sqrt{2m_p}} \quad \left[\begin{array}{l} m_\alpha = 4m_p \\ q_\alpha = 2q_p \end{array} \right]$$

$$= 1$$

Mass of electron is least and charge $q_e = e$

So, $r_e < r_p = r_\alpha$

12. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric

material of dielectric constant $K = \frac{5}{3}$ is inserted

between the plates, the magnitude of the induced charge will be

- (1) 2.4 nC (2) 0.9 nC
(3) 1.2 nC (4) 0.3 nC

Answer (3)

Sol. $C' = KC_0$

$Q = KC_0 V$

$Q_{\text{induced}} = Q \left(1 - \frac{1}{K} \right)$

$= \frac{5}{3} \times 90 \times 10^{-12} \times 20 \left(1 - \frac{3}{5} \right)$

$= 1.2 \text{ nC}$

13. For an RLC circuit driven with voltage of amplitude v_m

and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits

resonance. The quality factor, Q is given by

- (1) $\frac{R}{(\omega_0 C)}$
(2) $\frac{CR}{\omega_0}$
(3) $\frac{\omega_0 L}{R}$
(4) $\frac{\omega_0 R}{L}$

Answer (3)

Sol. Quality factor, $Q = \frac{\omega_0}{(2\Delta\omega)}$

$Q = \frac{\omega_0 L}{R}$

14. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

- (1) 2×10^5 (2) 2×10^6
(3) 2×10^3 (4) 2×10^4

Answer (1)

Sol. Frequency of carrier = $10 \times 10^9 \text{ Hz}$

Available bandwidth = 10% of $10 \times 10^9 \text{ Hz}$
 $= 10^9 \text{ Hz}$

Bandwidth for each telephonic channel = 5 kHz

$\therefore \text{Number of channels} = \frac{10^9}{5 \times 10^3}$
 $= 2 \times 10^5$

15. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 \text{ kg/m}^3$ and its Young's modulus is $9.27 \times 10^{10} \text{ Pa}$. What will be the fundamental frequency of the longitudinal vibrations?

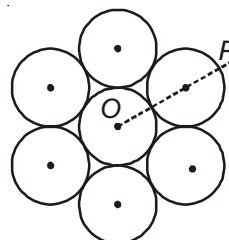
- (1) 10 kHz (2) 7.5 kHz
(3) 5 kHz (4) 2.5 kHz

Answer (3)

Sol. $f_0 = \frac{V}{2L} = \frac{1}{2L} \sqrt{\frac{Y}{\rho}}$

$= \frac{1}{2 \times 0.6} \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}} = 4.88 \text{ kHz} \approx 5 \text{ kHz}$

16. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is



- (1) $\frac{73}{2} MR^2$ (2) $\frac{181}{2} MR^2$
(3) $\frac{19}{2} MR^2$ (4) $\frac{55}{2} MR^2$

Answer (2)

Sol. $I_0 = \frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)$

$$I_P = I_0 + 7M(3R)^2$$

$$= \frac{181}{2}MR^2$$

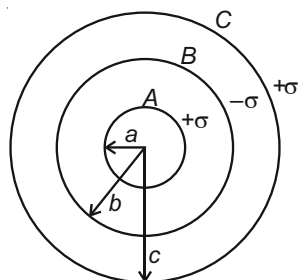
17. Three concentric metal shells A, B and C of respective radii a , b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$ respectively. The potential of shell B is

(1) $\frac{\sigma}{\epsilon_0} \left[\frac{b^2 - c^2}{b} + a \right]$ (2) $\frac{\sigma}{\epsilon_0} \left[\frac{b^2 - c^2}{c} + a \right]$

(3) $\frac{\sigma}{\epsilon_0} \left[\frac{a^2 - b^2}{a} + c \right]$ (4) $\frac{\sigma}{\epsilon_0} \left[\frac{a^2 - b^2}{b} + c \right]$

Answer (4)

Sol.



$$V_B = \left[\frac{\sigma 4\pi a^2}{4\pi\epsilon_0 b} - \frac{\sigma 4\pi b^2}{4\pi\epsilon_0 b} + \frac{\sigma 4\pi c^2}{4\pi\epsilon_0 c} \right]$$

$$V_B = \frac{\sigma}{\epsilon_0} \left[\frac{a^2 - b^2}{b} + c \right]$$

18. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5Ω , a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

- (1) 2Ω
(2) 2.5Ω
(3) 1Ω
(4) 1.5Ω

Answer (4)

Sol. $\therefore E \propto l_1$

and $E - ir \propto l_2$

$$\therefore \frac{E}{E - ir} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{E}{E - \left(\frac{E}{r+5}\right) \times r} = \frac{52}{40}$$

$$\Rightarrow \frac{r+5}{5} = \frac{13}{10}$$

$$\Rightarrow r = 1.5 \Omega$$

19. An EM wave from air enters a medium. The electric

fields are $\vec{E}_1 = E_{01} \hat{x} \cos \left[2\pi v \left(\frac{z}{c} - t \right) \right]$ in air and

$\vec{E}_2 = E_{02} \hat{x} \cos [k(2z - ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic. If ϵ_{r1} and ϵ_{r2} refer to relative permittivities of air and medium respectively, which of the following options is correct?

(1) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{4}$ (2) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = \frac{1}{2}$

(3) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = 4$ (4) $\frac{\epsilon_{r1}}{\epsilon_{r2}} = 2$

Answer (1)

Sol. $\vec{E}_1 = E_{01} \hat{x} \cos \left[2\pi v \left(\frac{z}{c} - t \right) \right]$ air

$\vec{E}_2 = E_{02} \hat{x} \cos [k(2z - ct)]$ medium

During refraction, frequency remains unchanged, whereas wavelength gets changed.

$$\therefore k' = 2k \quad (\text{From equations})$$

$$\Rightarrow \frac{2\pi}{\lambda'} = 2 \left(\frac{2\pi}{\lambda_0} \right)$$

$$\Rightarrow \lambda' = \frac{\lambda_0}{2}$$

$$\Rightarrow v = \frac{c}{2}$$

$$\Rightarrow \frac{1}{\sqrt{\mu_0 \epsilon_2}} = \frac{1}{2} \times \frac{1}{\sqrt{\mu_0 \epsilon_1}}$$

$$\Rightarrow \frac{\epsilon_1}{\epsilon_2} = \frac{1}{4}$$

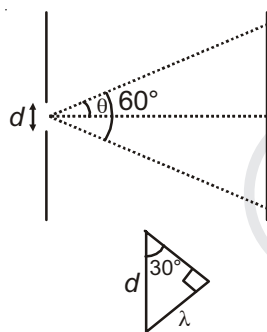
20. The angular width of the central maximum in a single slit diffraction pattern is 60° . The width of the slit is $1 \mu\text{m}$. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm , what is slit separation distance?

(i.e. distance between the centres of each slit.)

- (1) $75 \mu\text{m}$ (2) $100 \mu\text{m}$
 (3) $25 \mu\text{m}$ (4) $50 \mu\text{m}$

Answer (3)

Sol. $d \sin \theta = \lambda$



$$\lambda = \frac{d}{2} \quad [d = 1 \times 10^{-6} \text{ m}]$$

$$\Rightarrow \lambda = 5000 \text{ \AA}$$

$$\text{Fringe width, } B = \frac{\lambda D}{d'} \quad (d' \text{ is slit separation})$$

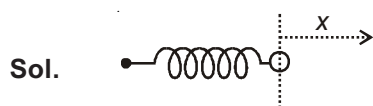
$$10^{-2} = \frac{5000 \times 10^{-10} \times 0.5}{d'}$$

$$\Rightarrow d' = 25 \times 10^{-6} \text{ m} = 25 \mu\text{m}$$

21. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}/\text{second}$. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = $6.02 \times 10^{23} \text{ gm mole}^{-1}$)

- (1) 2.2 N/m (2) 5.5 N/m
 (3) 6.4 N/m (4) 7.1 N/m

Answer (4)



Sol.

$$Kx = ma \Rightarrow a = (K/m)x$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{K}{m}} = 10^{12}$$

$$= \frac{1}{4\pi^2} \times \frac{K}{m} = 10^{24}$$

$$K = 4\pi^2 m \times 10^{24} = \frac{4 \times 10 \times 108 \times 10^{-3}}{6.02 \times 10^{23}} \times 10^{24}$$

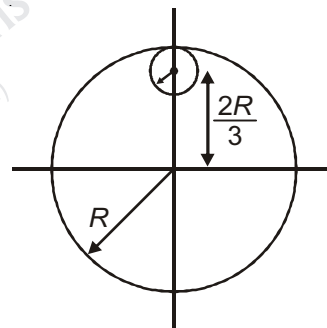
$$= 7.1 \text{ N/m}$$

22. From a uniform circular disc of radius R and mass

$9M$, a small disc of radius $\frac{R}{3}$ is removed as shown

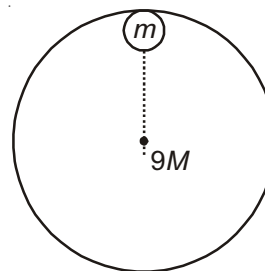
in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

- (1) $10MR^2$
 (2) $\frac{37}{9} MR^2$
 (3) $4MR^2$
 (4) $\frac{40}{9} MR^2$



Answer (3)

Sol.



$$m = \frac{(9M)}{9} = M$$

$$I_1 = \frac{(9M) \times R^2}{2}$$

$$I_2 = \frac{M \times \left(\frac{R}{3}\right)^2}{2} + M \times \left(\frac{2R}{3}\right)^2 = \frac{MR^2}{2}$$

$$\therefore I_{\text{req}} = I_1 - I_2$$

$$= \frac{9}{2} MR^2 - \frac{MR^2}{2}$$

$$= 4MR^2$$

23. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is

- (1) $\frac{v_0}{2}$ (2) $\frac{v_0}{\sqrt{2}}$
(3) $\frac{v_0}{4}$ (4) $\sqrt{2}v_0$

Answer (4)

Sol. It is a case of superelastic collision

$$mv_0 = mv_1 + mv_2 \quad \dots(i)$$

$$\Rightarrow v_1 + v_2 = v_0$$

$$\frac{1}{2}m(v_1^2 + v_2^2) = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow (v_1^2 + v_2^2) = \frac{3}{2}v_0^2 \quad \dots(ii)$$

$$\Rightarrow (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2$$

$$\Rightarrow v_0^2 = \frac{3v_0^2}{2} + 2v_1v_2$$

$$\Rightarrow 2v_1v_2 = -\frac{v_0^2}{2} \quad \dots(iii)$$

$$\therefore (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v_0^2 + v_0^2$$

$$\Rightarrow v_1 - v_2 = \sqrt{2}v_0$$

24. The dipole moment of a circular loop carrying a current I , is m and the magnetic field at the centre of the loop is B_1 . When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2 . The

ratio $\frac{B_1}{B_2}$ is

- (1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$
(3) 2 (4) $\sqrt{3}$

Answer (1)

Sol. $m = I(\pi R^2)$, $m' = 2m = I \times (\pi\sqrt{2}R)^2$

$$\therefore R' = \sqrt{2}R$$

$$B_1 = \frac{\mu_0 I}{2R}$$

$$B_2 = \frac{\mu_0 I}{2 \times (\sqrt{2}R)}$$

$$\therefore \frac{B_1}{B_2} = \sqrt{2}$$

25. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

- (1) 4.5% (2) 6%
(3) 2.5% (4) 3.5%

Answer (1)

Sol. $\rho = \frac{m}{l^3}$

$$\begin{aligned} \frac{d\rho}{\rho} &= \frac{dm}{m} + 3 \frac{dl}{l} \\ &= (1.5 + 3 \times 1) \\ &= 4.5\% \end{aligned}$$

26. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 k Ω . How much was the resistance on the left slot before interchanging the resistances?

- (1) 550 Ω
(2) 910 Ω
(3) 990 Ω
(4) 505 Ω

Answer (1)

$$\text{Sol. } \frac{R_1}{R_2} = \frac{l}{(100-l)}$$

$$\frac{R_2}{R_1} = \frac{(l-10)}{(110-l)}$$

$$(100-l)(110-l) = l(l-10)$$

$$11000 + l^2 - 210l = l^2 - 10l$$

$$\Rightarrow l = 55 \text{ cm}$$

$$R_1 = R_2 \left(\frac{55}{45} \right)$$

$$R_1 + R_2 = 1000 \Omega$$

$$R_1 = 550 \Omega$$

27. In an a.c. circuit, the instantaneous e.m.f. and current are given by

$$e = 100 \sin 30t$$

$$i = 20 \sin \left(30t - \frac{\pi}{4} \right)$$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively

- (1) $\frac{50}{\sqrt{2}}, 0$
 (2) 50, 0
 (3) 50, 10
 (4) $\frac{1000}{\sqrt{2}}, 10$

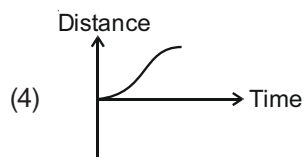
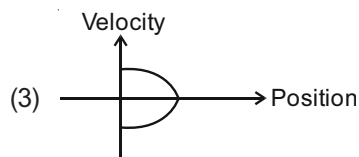
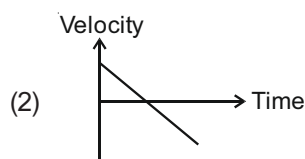
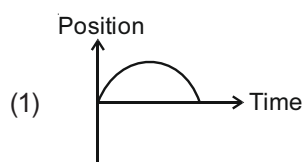
Answer (4)

Sol. $P_{av} = E_{rms} I_{rms} \cos \phi$

$$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$$

$$i_{wattless} = i_{rms} \sin \phi = \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 10$$

28. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



Answer (4)

Sol. Options (1), (2) and (3) correspond to uniformly accelerated motion in a straight line with positive initial velocity and constant negative acceleration, whereas option (4) does not correspond to this motion.

29. Two moles of an ideal monoatomic gas occupies a volume V at 27°C . The gas expands adiabatically to a volume $2V$. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

- (1) (a) 189 K (b) -2.7 kJ
 (2) (a) 195 K (b) 2.7 kJ
 (3) (a) 189 K (b) 2.7 kJ
 (4) (a) 195 K (b) -2.7 kJ

Answer (1)

Sol. $TV^{\gamma-1} = \text{Constant}$

$$T_f = 300 \left(\frac{V}{2V} \right)^{\frac{5}{3}-1} = 189 \text{ K}$$

$$\Delta U = nC_v \Delta T = 2 \times \frac{3R}{2} \times [189 - 300]$$

$$= -2.7 \text{ kJ}$$

30. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R . If the period of rotation of the particle is T , then

- (1) $T \propto R^{(n+1)/2}$
 (2) $T \propto R^{n/2}$
 (3) $T \propto R^{3/2}$ for any n
 (4) $T \propto R^{\frac{n}{2}+1}$

Answer (1)

Sol. $m\omega^2 R = k R^{-n} = \frac{k}{R^n}$

$$\Rightarrow \frac{1}{T^2} \propto \frac{1}{R^{n+1}}$$

$$\Rightarrow T \propto R^{\left(\frac{n+1}{2}\right)}$$

PART-B : MATHEMATICS

31. If the tangent at $(1, 7)$ to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is

- (1) 85
(2) 95
(3) 195
(4) 185

Answer (2)

Sol. Equation of tangent at $(1, 7)$ to curve $x^2 = y - 6$ is

$$x - 1 = \frac{1}{2}(y + 7) - 6$$

$$2x - y + 5 = 0 \quad \dots(i)$$

$$\text{Centre of circle} = (-8, -6)$$

$$\text{Radius of circle} = \sqrt{64 + 36 - c} = \sqrt{100 - c}$$

$\therefore$ Line (i) touches the circle

$$\therefore \left| \frac{2(-8) - (-6) + 5}{\sqrt{4 + 1}} \right| = \sqrt{100 - c}$$

$$\sqrt{5} = \sqrt{100 - c}$$

$$\Rightarrow c = 95$$

32. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines L_1 and L_2 , is

- (1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$
(3) $\frac{1}{4\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

Answer (4)

Sol. L_1 is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i} + \hat{j}$

$$L_2 \text{ is parallel to } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix} = 3\hat{i} - 5\hat{j} - 7\hat{k}$$

Also, L_2 passes through $\left(\frac{5}{7}, \frac{8}{7}, 0\right)$

So, required plane is $\begin{vmatrix} x - \frac{5}{7} & y - \frac{8}{7} & z \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = 0$

$$\Rightarrow 7x - 7y + 8z + 3 = 0$$

Now, perpendicular distance $= \frac{3}{\sqrt{162}} = \frac{1}{3\sqrt{2}}$

33. If $\alpha, \beta \in \mathbb{C}$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

- (1) 1 (2) 2
(3) -1 (4) 0

Answer (1)

Sol. $x^2 - x + 1 = 0$

Roots are $-\omega, -\omega^2$

Let $\alpha = -\omega, \beta = -\omega^2$

$$\begin{aligned} \alpha^{101} + \beta^{107} &= (-\omega)^{101} + (-\omega^2)^{107} \\ &= -(\omega^{101} + \omega^{214}) \\ &= -(\omega^2 + \omega) \\ &= 1 \end{aligned}$$

34. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q . If these tangents intersect at the point $T(0, 3)$ then the area (in sq. units) of ΔPTQ is

- (1) $60\sqrt{3}$ (2) $36\sqrt{5}$
(3) $45\sqrt{5}$ (4) $54\sqrt{3}$

Answer (3)

Sol. Clearly PQ is a chord of contact,

i.e., equation of PQ is $T \equiv 0$

$$\Rightarrow y = -12$$

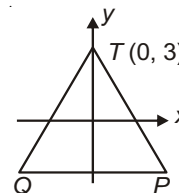
Solving with the curve, $4x^2 - y^2 = 36$

$$\Rightarrow x = \pm 3\sqrt{5}, y = -12$$

i.e., $P(3\sqrt{5}, -12); Q(-3\sqrt{5}, -12); T(0, 3)$

Area of ΔPQT is

$$\begin{aligned} \Delta &= \frac{1}{2} \times 6\sqrt{5} \times 15 \\ &= 45\sqrt{5} \end{aligned}$$



35. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

- (1) 4 (2) $\frac{9}{2}$
 (3) 6 (4) $\frac{7}{2}$

Answer (2)

Sol. $y^2 = 6x$; slope of tangent at (x_1, y_1) is $m_1 = \frac{3}{y_1}$

also $9x^2 + by^2 = 16$; slope of tangent at (x_1, y_1) is

$$m_2 = \frac{-9x_1}{by_1}$$

$$\text{As } m_1 m_2 = -1$$

$$\Rightarrow \frac{-27x_1}{by_1^2} = -1$$

$$\Rightarrow b = \frac{9}{2} \text{ (as } y_1^2 = 6x_1)$$

36. If the system of linear equations

$$x + ky + 3z = 0$$

$$3x + ky - 2z = 0$$

$$2x + 4y - 3z = 0$$

has a non-zero solution (x, y, z) , then $\frac{xz}{y^2}$ is equal to

- (1) -30 (2) 30
 (3) -10 (4) 10

Answer (4)

Sol. $\therefore$ System of equation has non-zero solution.

$$\therefore \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$$\Rightarrow 44 - 4k = 0$$

$$\therefore k = 11$$

$$\text{Let } z = \lambda$$

$$\therefore x + 11y = -3\lambda$$

$$\text{and } 3x + 11y = 2\lambda$$

$$\therefore x = \frac{5\lambda}{2}, y = -\frac{\lambda}{2}, z = \lambda$$

$$\therefore \frac{xz}{y^2} = \frac{\frac{5\lambda}{2} \cdot \lambda}{\left(-\frac{\lambda}{2}\right)^2} = 10$$

37. Let $S = \{x \in R : x \geq 0 \text{ and}$

$$2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$$
. Then S :

- (1) Contains exactly two elements
 (2) Contains exactly four elements
 (3) Is an empty set
 (4) Contains exactly one element

Answer (1)

Sol. $2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0$

$$2|\sqrt{x} - 3| + (\sqrt{x} - 3 + 3)(\sqrt{x} - 3 - 3) + 6 = 0$$

$$2|\sqrt{x} - 3| + (\sqrt{x} - 3)^2 - 3 = 0$$

$$(\sqrt{x} - 3)^2 + 2|\sqrt{x} - 3| - 3 = 0$$

$$(|\sqrt{x} - 3| + 3)(|\sqrt{x} - 3| - 1) = 0$$

$$\Rightarrow |\sqrt{x} - 3| = 1, |\sqrt{x} - 3| + 3 \neq 0$$

$$\Rightarrow \sqrt{x} - 3 = \pm 1$$

$$\Rightarrow \sqrt{x} = 4, 2$$

$$x = 16, 4$$

38. If sum of all the solutions of the equation

$$8 \cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2} \right) = 1 \text{ in } [0, \pi]$$

is $k\pi$, then k is equal to :

- (1) $\frac{8}{9}$ (2) $\frac{20}{9}$
 (3) $\frac{2}{3}$ (4) $\frac{13}{9}$

Answer (4)

$$\text{Sol. } 8 \cos x \cdot \left(\cos^2 \frac{\pi}{6} - \sin^2 x - \frac{1}{2} \right) = 1$$

$$\Rightarrow 8 \cos x \left(\frac{3}{4} - \frac{1}{2} - 1 + \cos^2 x \right) = 1$$

$$\Rightarrow 8 \cos x \left(\frac{-3 + 4 \cos^2 x}{4} \right) = 1$$

$$\Rightarrow \cos 3x = 1$$

$$\Rightarrow \cos 3x = \frac{1}{2}$$

$$\Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

$$\Rightarrow \text{Sum} = \frac{13\pi}{9}$$

$$\Rightarrow k = \frac{13}{9}$$

39. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(1) $\frac{1}{5}$ (2) $\frac{3}{4}$

(3) $\frac{3}{10}$ (4) $\frac{2}{5}$

Answer (4)

Sol. E_1 : Event that first ball drawn is red.

E_2 : Event that first ball drawn is black.

E : Event that second ball drawn is red.

$$P(E) = P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)$$

$$= \frac{4}{10} \times \frac{6}{12} + \frac{6}{10} \times \frac{4}{12} = \frac{2}{5}$$

40. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in \mathbb{R} - \{-1, 0, 1\}$.

If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is:

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$
(3) 3 (4) -3

Answer (2)

Sol. $h(x) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}}$

$$= \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)}$$

$$x - \frac{1}{x} > 0, \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)} \in (2\sqrt{2}, \infty]$$

$$x - \frac{1}{x} < 0, \left(x - \frac{1}{x}\right) + \frac{2}{\left(x - \frac{1}{x}\right)} \in (-\infty, -2\sqrt{2}]$$

Local minimum is $2\sqrt{2}$

41. Two sets A and B are as under :

$$A = \{(a, b) \in \mathbb{R} \times \mathbb{R} : |a - 5| < 1 \text{ and } |b - 5| < 1\}$$

$$B = \{(a, b) \in \mathbb{R} \times \mathbb{R} : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\},$$

then

(1) $A \cap B = \phi$ (an empty set)

(2) Neither $A \subset B$ nor $B \subset A$

(3) $B \subset A$

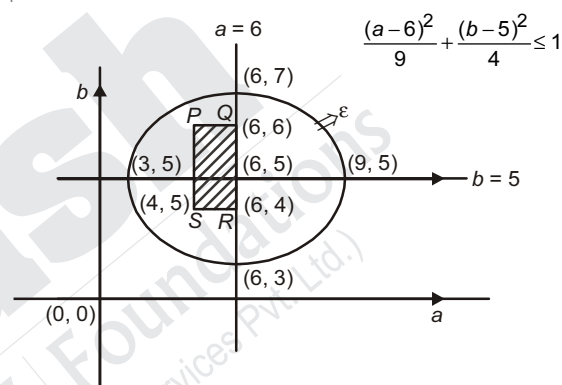
(4) $A \subset B$

Answer (4)

Sol. As, $|a - 5| < 1$ and $|b - 5| < 1$

$$\Rightarrow 4 < a, b < 6 \text{ and } \frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \leq 1$$

Taking axes as a -axis and b -axis



The set A represents square $PQRS$ inside set B representing ellipse and hence $A \subset B$.

42. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

(1) q (2) $\sim q$
(3) $\sim p$ (4) p

Answer (3)

Sol. $\sim(p \vee q) \vee (\sim p \wedge q)$

By property, $(\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$$= \sim p$$

43. Tangent and normal are drawn at $P(16, 16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B , respectively. If C is the centre of the circle through the points P , A and B and $\angle CPB = \theta$, then a value of $\tan \theta$ is

(1) 3 (2) $\frac{4}{3}$
(3) $\frac{1}{2}$ (4) 2

Answer (4)

Sol. $y^2 = 16x$

Tangent at $P(16, 16)$ is $2y = x + 16$... (1)

Normal at $P(16, 16)$ is $y = -2x + 48$... (2)

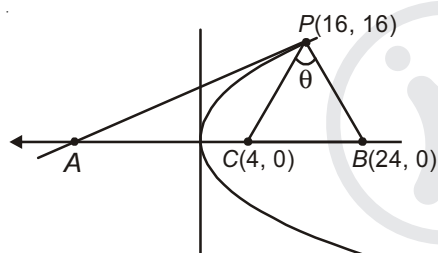
i.e., A is $(-16, 0)$; B is $(24, 0)$

Now, Centre of circle is $(4, 0)$

$$\text{Now, } m_{PC} = \frac{4}{3}$$

$$m_{PB} = -2$$

$$\text{i.e., } \tan \theta = \left| \frac{\frac{4}{3} + 2}{1 - \frac{8}{3}} \right| = 2$$



44. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the

ordered pair (A, B) is equal to

(1) $(-4, 5)$

(2) $(4, 5)$

(3) $(-4, -5)$

(4) $(-4, 3)$

Answer (1)

Sol. $\Delta = \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$

$x = -4$ makes all three row identical

hence $(x + 4)^2$ will be factor

Also, $C_1 \rightarrow C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

$\Rightarrow 5x - 4$ is a factor

$$\Delta = \lambda(5x-4)(x+4)^2$$

$$\therefore B = 5, A = -4$$

45. The sum of the co-efficients of all odd degree terms in the expansion of $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$, ($x > 1$) is

(1) 1

(2) 2

(3) -1

(4) 0

Answer (2)

Sol. $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$

$$= 2[{}^5C_0 x^5 + {}^5C_2 x^3 (x^3 - 1) + {}^5C_4 x (x^3 - 1)^2]$$

$$= 2[x^5 + 10(x^6 - x^3) + 5x(x^6 - 2x^3 + 1)]$$

$$= 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$$

$$= 2[5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x]$$

Sum of odd degree terms coefficients

$$= 2(5 + 1 - 10 + 5)$$

$$= 2$$

46. Let $a_1, a_2, a_3, \dots, a_{49}$ be in A.P. such that

$$\sum_{k=0}^{12} a_{4k+1} = 416 \text{ and } a_9 + a_{43} = 66.$$

If $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$, then m is equal to

(1) 34

(2) 33

(3) 66

(4) 68

Answer (1)

Sol. Let $a_1 = a$ and common difference = d

$$\text{Given, } a_1 + a_5 + a_9 + \dots + a_{49} = 416$$

$$\Rightarrow a + 24d = 32 \quad \dots(i)$$

$$\text{Also, } a_9 + a_{43} = 66 \Rightarrow a + 25d = 33 \quad \dots(ii)$$

Solving (i) & (ii),

We get $d = 1, a = 8$

$$\text{Now, } a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$$

$$\Rightarrow 8^2 + 9^2 + \dots + 24^2 = 140m$$

$$\Rightarrow \frac{24 \times 25 \times 49}{6} - \frac{7 \times 8 \times 15}{6} = 140m$$

$$\Rightarrow \boxed{m = 34}$$

47. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

- (1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$
(3) $3x + 2y = 6$ (4) $2x + 3y = xy$

Answer (1)

Sol. Let the equation of line be $\frac{x}{a} + \frac{y}{b} = 1$... (i)

(i) passes through the fixed point (2, 3)

$$\Rightarrow \frac{2}{a} + \frac{3}{b} = 1 \quad \dots (ii)$$

P(a, 0), Q(0, b), O(0, 0), Let R(h, k),
Q(0, b) R(h, k)



O(0, 0) P(a, 0)

Midpoint of OR is $\left(\frac{h}{2}, \frac{k}{2}\right)$

Midpoint of PQ is $\left(\frac{a}{2}, \frac{b}{2}\right) \Rightarrow h = a, k = b \dots (iii)$

From (ii) & (iii),

$$\frac{2}{h} + \frac{3}{k} = 1 \Rightarrow \text{locus of } R(h, k)$$

$$\frac{2}{x} + \frac{3}{y} = 1 \Rightarrow 3x + 2y = xy$$

48. Then value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is :

- (1) 4π
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{8}$
(4) $\frac{\pi}{2}$

Answer (2)

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx \quad \dots (i)$

$$\text{Also, } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \sin^2 x}{1+2^x} dx \quad \dots (ii)$$

Adding (i) and (ii)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x dx$$

$$2I = 2 \int_0^{\frac{\pi}{2}} \sin^2 x dx \Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \quad \dots (iii)$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x dx \quad \dots (iv)$$

Adding (iii) & (iv)

$$2I = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

49. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (g \circ f)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$, is

- (1) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$ (2) $\frac{1}{2}(\sqrt{2} - 1)$
(3) $\frac{1}{2}(\sqrt{3} - 1)$ (4) $\frac{1}{2}(\sqrt{3} + 1)$

Answer (3)

Sol. $18x^2 - 9\pi x + \pi^2 = 0$

$$(6x - \pi)(3x - \pi) = 0$$

$$\therefore x = \frac{\pi}{6}, \frac{\pi}{3}$$

$$\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$$

$$y = (g \circ f)(x) = \cos x$$

$$\begin{aligned} \text{Area} &= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x dx = (\sin x)_{\frac{\pi}{6}}^{\frac{\pi}{3}} \\ &= \frac{\sqrt{3}}{2} - \frac{1}{2} \\ &= \frac{1}{2}(\sqrt{3} - 1) \text{ sq. units} \end{aligned}$$

50. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t . Then

$$\lim_{x \rightarrow 0^+} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{15}{x} \right] \right)$$

- (1) Is equal to 120 (2) Does not exist (in R)
 (3) Is equal to 0 (4) Is equal to 15

Answer (1)

Sol. As $\frac{1}{x} - 1 < \left[\frac{1}{x} \right] \leq \frac{1}{x}$

$$\frac{2}{x} - 1 < \left[\frac{2}{x} \right] \leq \frac{2}{x}$$

$$\sum_{r=1}^{15} \left(\frac{r}{x} - 1 \right) < \sum_{r=1}^{15} \left(\frac{r}{x} \right) \leq \sum_{r=1}^{15} \frac{r}{x}$$

$$120 < \lim_{x \rightarrow 0^+} x \left(\sum_{r=1}^{15} \left[\frac{r}{x} \right] \right) \leq 120$$

$$\Rightarrow \lim_{x \rightarrow 0^+} x \left(\left[\frac{1}{x} \right] + \left[\frac{2}{x} \right] + \dots + \left[\frac{15}{x} \right] \right) = 120$$

51. If $\sum_{i=1}^9 (x_i - 5) = 9$ and $\sum_{i=1}^9 (x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \dots, x_9$ is

- (1) 2 (2) 3
 (3) 9 (4) 4

Answer (1)

Sol. Standard deviation of $x_i - 5$ is

$$\sigma = \sqrt{\frac{\sum_{i=1}^9 (x_i - 5)^2}{9} - \left(\frac{\sum_{i=1}^9 (x_i - 5)}{9} \right)^2}$$

$$\Rightarrow \sigma = \sqrt{5 - 1} = 2$$

As, standard deviation remains constant if observations are added/subtracted by a fixed quantity.

So, σ of x_i is 2

52. The integral

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

is equal to

- (1) $\frac{1}{1 + \cot^3 x} + C$ (2) $\frac{-1}{1 + \cot^3 x} + C$
 (3) $\frac{1}{3(1 + \tan^3 x)} + C$ (4) $\frac{-1}{3(1 + \tan^3 x)} + C$

(where C is a constant of integration)

Answer (4)

$$\text{Sol. } I = \int \frac{\sin^2 x \cos^2 x dx}{\{(\sin^2 x + \cos^2 x)(\sin^3 x + \cos^3 x)\}^2}$$

Dividing the numerator and denominator by $\cos^6 x$

$$\Rightarrow I = \int \frac{\tan^2 x \sec^2 x dx}{(1 + \tan^3 x)^2}$$

Let, $\tan^3 x = z$

$$\Rightarrow 3 \tan^2 x \cdot \sec^2 x dx = dz$$

$$I = \frac{1}{3} \int \frac{dz}{z^2} = \frac{-1}{3z} + C$$

$$= \frac{-1}{3(1 + \tan^3 x)} + C$$

53. Let $S = \{t \in R : f(x) = |x - \pi| \cdot (e^{|x|} - 1) \sin |x| \text{ is not differentiable at } t\}$. Then the set S is equal to

- (1) $\{\pi\}$ (2) $\{0, \pi\}$
 (3) ϕ (an empty set) (4) $\{0\}$

Answer (3)

$$\text{Sol. } f(x) = |x - \pi| (e^{|x|} - 1) \sin |x|$$

$x = \pi, 0$ are repeated roots and also continuous.

Hence, ' f ' is differentiable at all x .

54. let $y = y(x)$ be the solution of the differential equation

$$\sin x \frac{dy}{dx} + y \cos x = 4x, \quad x \in (0, \pi). \text{ If } y = \left(\frac{\pi}{2} \right) = 0,$$

then $y\left(\frac{\pi}{6}\right)$ is equal to :

- (1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$
 (3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $\frac{-8}{9\sqrt{3}}\pi^2$

Answer (1)

Sol. $\sin x \frac{dy}{dx} + y \cos x = 4x, x \in (0, \pi)$

$$\frac{dy}{dx} + y \cot x = \frac{4x}{\sin x}$$

$$\therefore \text{I.F.} = e^{\int \cot x \, dx} = \sin x$$

$\therefore$ Solution is given by

$$y \sin x = \int \frac{4x}{\sin x} \cdot \sin x \, dx$$

$$y \cdot \sin x = 2x^2 + c$$

$$\text{when } x = \frac{\pi}{2}, y = 0 \Rightarrow c = -\frac{\pi^2}{2}$$

$$\therefore \text{Equation is : } y \sin x = 2x^2 - \frac{\pi^2}{2}$$

$$\text{when } x = \frac{\pi}{6} \text{ then } y \cdot \frac{1}{2} = 2 \cdot \frac{\pi^2}{36} - \frac{\pi^2}{2}$$

$$\therefore y = -\frac{8\pi^2}{9}$$

55. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to

- (1) 256
(2) 84
(3) 336
(4) 315

Answer (3)

Sol. Clearly, $\vec{u} = \lambda(\vec{a} \times (\vec{a} \times \vec{b}))$

$$\Rightarrow \vec{u} = \lambda((\vec{a} \cdot \vec{b})\vec{a} - |\vec{a}|^2 \vec{b})$$

$$\Rightarrow \vec{u} = \lambda(2\vec{a} - 14\vec{b}) = 2\lambda\{(2\hat{i} + 3\hat{j} - \hat{k}) - 7(\hat{j} + \hat{k})\}$$

$$\Rightarrow \vec{u} = 2\lambda(2\hat{i} - 4\hat{j} - 8\hat{k})$$

$$\text{as, } \vec{u} \cdot \vec{b} = 24$$

$$\Rightarrow 4\lambda(\hat{i} - 2\hat{j} - 4\hat{k}) \cdot (\hat{j} + \hat{k}) = 24$$

$$\Rightarrow \lambda = -1$$

$$\text{So, } \vec{u} = -4(\hat{i} - 2\hat{j} - 4\hat{k})$$

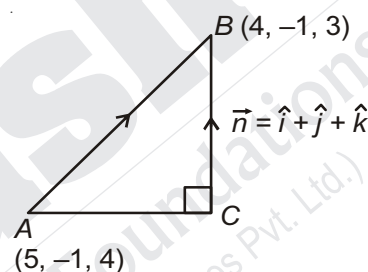
$$\Rightarrow |\vec{u}|^2 = 336$$

56. The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane, $x + y + z = 7$ is:

- (1) $\frac{1}{3}$
(2) $\sqrt{\frac{2}{3}}$
(3) $\frac{2}{\sqrt{3}}$
(4) $\frac{2}{3}$

Answer (2)

Sol.



Normal to the plane $x + y + z = 7$ is $\vec{n} = \hat{i} + \hat{j} + \hat{k}$

$$\vec{AB} = -\hat{i} - \hat{k} \Rightarrow |\vec{AB}| = AB = \sqrt{2}$$

$$BC = \text{Length of projection of } \vec{AB} \text{ on } \vec{n} = |\vec{AB} \cdot \hat{n}|$$

$$= \left| (-\hat{i} - \hat{k}) \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} \right| = \frac{2}{\sqrt{3}}$$

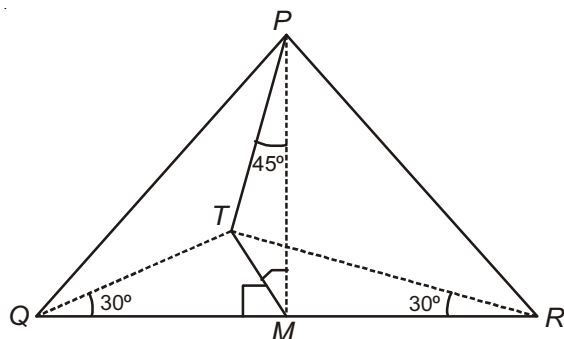
Length of projection of the line segment on the plane is AC

$$AC^2 = AB^2 - BC^2 = 2 - \frac{4}{3} = \frac{2}{3}$$

$$AC = \sqrt{\frac{2}{3}}$$

57. PQR is a triangular park with $PQ = PR = 200$ m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45° , 30° and 30° , then the height of the tower (in m) is

- (1) $100\sqrt{3}$
(2) $50\sqrt{2}$
(3) 100
(4) 50

Answer (3)**Sol.**Let height of tower TM be h

$$\therefore PM = h$$

$$\text{In } \triangle TQM, \quad \tan 30^\circ = \frac{h}{QM}$$

$$QM = \sqrt{3}h$$

$$\text{In } \triangle PMQ, \quad PM^2 + QM^2 = PQ^2$$

$$h^2 + (\sqrt{3}h)^2 = 200^2$$

$$\Rightarrow 4h^2 = 200^2$$

$$\Rightarrow h = 100 \text{ m}$$

58. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

- (1) At least 500 but less than 750
- (2) At least 750 but less than 1000
- (3) At least 1000
- (4) Less than 500

Answer (3)

Sol. Number of ways of selecting 4 novels from 6 novels
 $= {}^6C_4$

Number of ways of selecting 1 dictionary from 3 dictionaries $= {}^3C_1$

$$\text{Required arrangements} = {}^6C_4 \times {}^3C_1 \times 4! = 1080$$

$$\Rightarrow \text{At least 1000}$$

59. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \dots$$

If $B - 2A = 100\lambda$, then λ is equal to

- (1) 464
- (2) 496
- (3) 232
- (4) 248

Answer (4)

$$\text{Sol. } A = 1^2 + 2.2^2 + 3^2 + \dots + 2.20^2$$

$$= (1^2 + 2^2 + 3^2 + \dots + 20^2) + 4(1^2 + 2^2 + 3^2 + \dots + 10^2)$$

$$= \frac{20 \times 21 \times 41}{6} + \frac{4 \times 10 \times 11 \times 21}{6}$$

$$= 2870 + 1540 = 4410$$

$$B = 1^2 + 2.2^2 + 3^2 + \dots + 2.40^2$$

$$= (1^2 + 2^2 + 3^2 + \dots + 40^2) + 4(1^2 + 2^2 + 3^2 + \dots + 20^2)$$

$$= \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6}$$

$$= 22140 + 11480 = 33620$$

$$\Rightarrow B - 2A = 33620 - 8820 = 24800$$

$$\Rightarrow 100\lambda = 24800$$

$$\lambda = 248$$

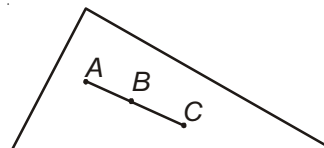
60. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

$$(1) 3\sqrt{\frac{5}{2}}$$

$$(2) \frac{3\sqrt{5}}{2}$$

$$(3) \sqrt{10}$$

$$(4) 2\sqrt{10}$$

Answer (1)**Sol.** $A(-3, 5)$ $B(3, 3)$ 

$$\text{So, } AB = 2\sqrt{10}$$

$$\text{Now, as, } AC = \frac{3}{2}AB$$

$$\text{So, radius} = \frac{3}{4}AB = \frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$$

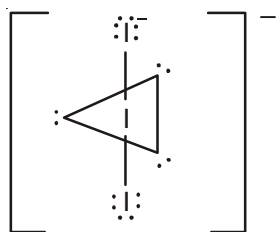
PART-C : CHEMISTRY

61. Total number of lone pair of electrons in I_3^- ion is

- (1) 9 (2) 12
(3) 3 (4) 6

Answer (1)

Sol. Structure of I_3^-



Number of lone pairs in I_3^- is 9.

62. Which of the following salts is the most basic in aqueous solution?

- (1) $FeCl_3$ (2) $Pb(CH_3COO)_2$
(3) $Al(CN)_3$ (4) CH_3COOK

Answer (4)

Sol. $CH_3COOK + H_2O \longrightarrow CH_3COOH + KOH$
Basic

$FeCl_3$ – Acidic solution

$Al(CN)_3$ – Salt of weak acid and weak base

$Pb(CH_3COO)_2$ – Salt of weak acid and weak base

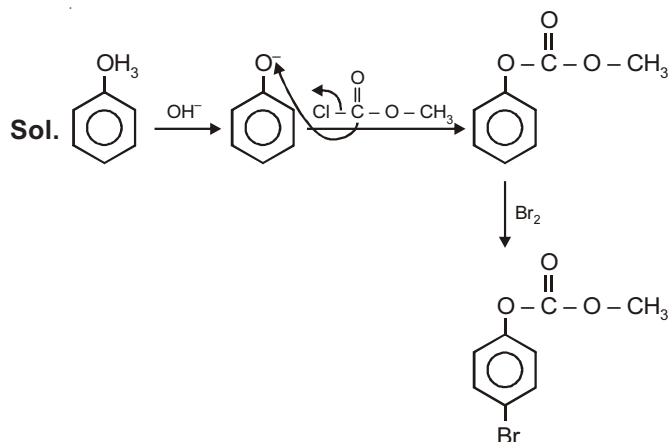
CH_3COOK is salt of weak acid and strong base.

Hence solution of CH_3COOK is basic.

63. Phenol reacts with methyl chloroformate in the presence of NaOH to form product A. A reacts with Br_2 to form product B. A and B are respectively

- (1) and
(2) and
(3) and
(4) and

Answer (1)



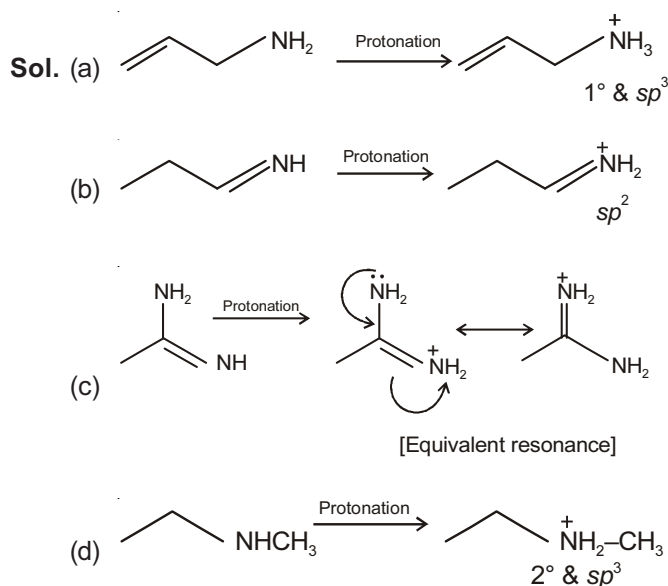
Hence, option (1) is correct.

64. The increasing order of basicity of the following compound is

- (a)
(b)
(c)
(d)

- (1) (b) < (a) < (d) < (c) (2) (d) < (b) < (a) < (c)
(3) (a) < (b) < (c) < (d) (4) (b) < (a) < (c) < (d)

Answer (1)



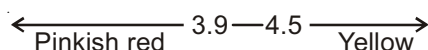
∴ Correct order of basicity : b < a < d < c.

65. An alkali is titrated against an acid with methyl orange as indicator, which of the following is a correct combination?

Base	Acid	End point
(1) Weak	Strong	Yellow to pinkish red
(2) Strong	Strong	Pink to colourless
(3) Weak	Strong	Colourless to pink
(4) Strong	Strong	Pinkish red to yellow

Answer (1)

Sol. The pH range of methyl orange is

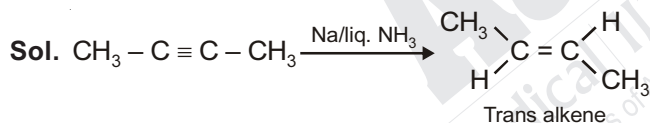


Weak base is having pH greater than 7. When methyl orange is added to weak base solution, the solution becomes yellow. This solution is titrated by strong acid and at the end point pH will be less than 3.1. Therefore solution becomes pinkish red.

66. The *trans*-alkenes are formed by the reduction of alkynes with

- (1) Na/liq. NH_3 (2) Sn - HCl
(3) H_2 - Pd/C, BaSO_4 (4) NaBH_4

Answer (1)



So, option (1) is correct.

67. The ratio of mass percent of C and H of an organic compound ($\text{C}_x\text{H}_y\text{O}_z$) is 6 : 1. If one molecule of the above compound ($\text{C}_x\text{H}_y\text{O}_z$) contains half as much oxygen as required to burn one molecule of compound C_xH_y completely to CO_2 and H_2O . The empirical formula of compound $\text{C}_x\text{H}_y\text{O}_z$ is

- (1) $\text{C}_3\text{H}_4\text{O}_2$ (2) $\text{C}_2\text{H}_4\text{O}_3$
(3) $\text{C}_3\text{H}_6\text{O}_3$ (4) $\text{C}_2\text{H}_4\text{O}$

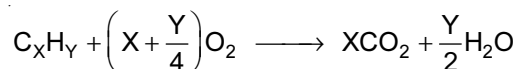
Answer (2)

Sol.

Element	Relative mass	Relative mole	Simplest whole number ratio
C	6	$\frac{6}{12} = 0.5$	1
H	1	$\frac{1}{1} = 1$	2

So, X = 1, Y = 2

Equation for combustion of C_xH_y



$$\text{Oxygen atoms required} = 2\left(x + \frac{y}{4}\right)$$

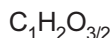
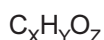
As per information,

$$2\left(x + \frac{y}{4}\right) = 2Z$$

$$\Rightarrow \left(1 + \frac{2}{4}\right) = Z$$

$$\Rightarrow Z = 1.5$$

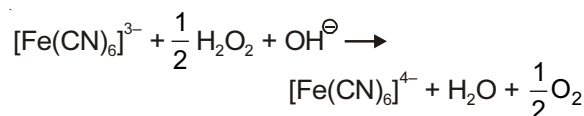
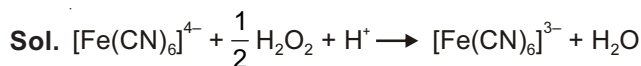
Molecule can be written



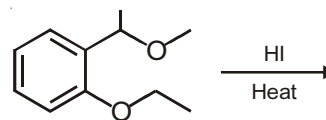
68. Hydrogen peroxide oxidises $[\text{Fe}(\text{CN})_6]^{4-}$ to $[\text{Fe}(\text{CN})_6]^{3-}$ in acidic medium but reduces $[\text{Fe}(\text{CN})_6]^{3-}$ to $[\text{Fe}(\text{CN})_6]^{4-}$ in alkaline medium. The other products formed are, respectively.

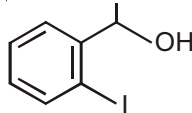
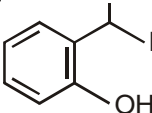
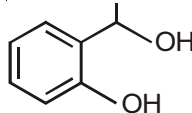
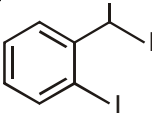
- (1) H_2O and $(\text{H}_2\text{O} + \text{O}_2)$
(2) H_2O and $(\text{H}_2\text{O} + \text{OH}^-)$
(3) $(\text{H}_2\text{O} + \text{O}_2)$ and H_2O
(4) $(\text{H}_2\text{O} + \text{O}_2)$ and $(\text{H}_2\text{O} + \text{OH}^-)$

Answer (1)

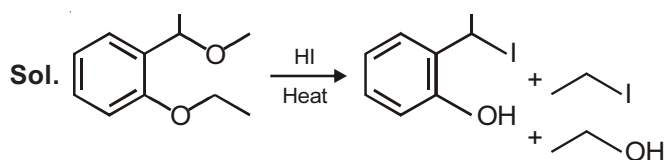


69. The major product formed in the following reaction is



- (1)  (2) 
(3)  (4) 

Answer (2)

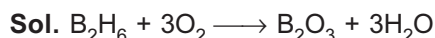


Hence, option (2) is correct.

70. How long (approximate) should water be electrolysed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

- (1) 3.2 hours
(2) 1.6 hours
(3) 6.4 hours
(4) 0.8 hours

Answer (1)



27.66 of $B_2H_6 = 1$ mole of B_2H_6 which requires three moles of oxygen (O_2) for complete burning



Number of faradays = 12 = Amount of charge

$$12 \times 96500 = i \times t$$

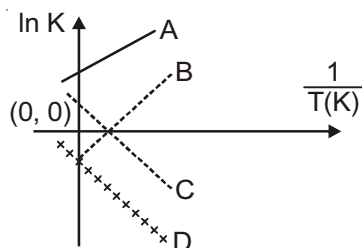
$$12 \times 96500 = 100 \times t$$

$$t = \frac{12 \times 96500}{100} \text{ second}$$

$$t = \frac{12 \times 96500}{100 \times 3600} \text{ hour}$$

$$t = 3.2 \text{ hours}$$

71. Which of the following lines correctly show the temperature dependence of equilibrium constant K, for an exothermic reaction?



- (1) C and D
(2) A and D
(3) A and B
(4) B and C

Answer (3)

Sol. Equilibrium constant $K = \left(\frac{A_f}{A_b}\right) e^{\frac{-\Delta H^\circ}{RT}}$

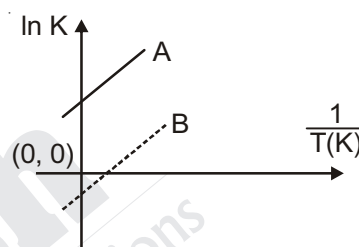
$$\ln K = \ln \left(\frac{A_f}{A_b}\right) - \frac{\Delta H^\circ}{R} \left(\frac{1}{T}\right)$$

$$y = C + m x$$

Comparing with equation of straight line,

$$\text{Slope} = \frac{-\Delta H^\circ}{R}$$

Since, reaction is exothermic, $\Delta H^\circ = -ve$, therefore, slope = +ve.



Hence, option (4) is correct.

72. At 518°C , the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 torr, was 1.00 torr s^{-1} when 5% had reacted and 0.5 torr s^{-1} when 33% had reacted. The order of the reaction is

- (1) 1 (2) 0
(3) 2 (4) 3

Answer (3)

Sol. Assume the order of reaction with respect to acetaldehyde is x.

Condition-1 :

$$\text{Rate} = k[\text{CH}_3\text{CHO}]^x$$

$$1 = k[363 \times 0.95]^x$$

$$1 = k[344.85]^x \quad \dots(i)$$

Condition-2 :

$$0.5 = k[363 \times 0.67]^x$$

$$0.5 = k[243.21]^x \quad \dots(ii)$$

Divide equation (i) by (ii),

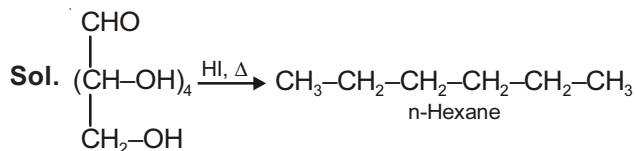
$$\frac{1}{0.5} = \left(\frac{344.85}{243.21}\right)^x \Rightarrow 2 = (1.414)^x$$

$$\Rightarrow x = 2$$

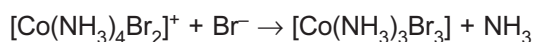
73. Glucose on prolonged heating with HI gives

- (1) Hexanoic acid (2) 6-iodohexanal
(3) *n*-Hexane (4) 1-Hexene

Answer (3)



74. Consider the following reaction and statements



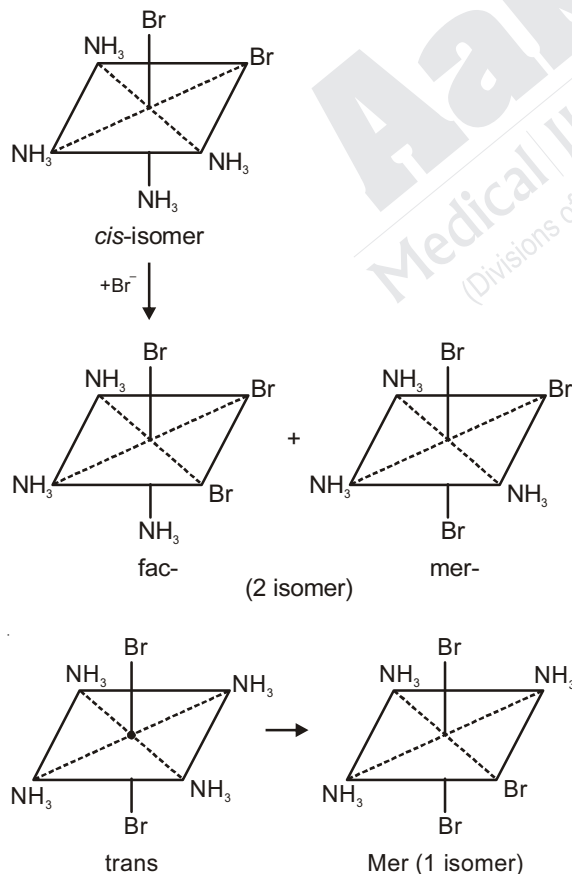
- (I) Two isomers are produced if the reactant complex ion is a *cis*-isomer
- (II) Two isomers are produced if the reactant complex ion is a *trans*-isomer.
- (III) Only one isomer is produced if the reactant complex ion is a *trans*-isomer.
- (IV) Only one isomer is produced if the reactant complex ion is a *cis*-isomer.

The correct statements are:

- (1) (III) and (IV) (2) (II) and (IV)
(3) (I) and (II) (4) (I) and (III)

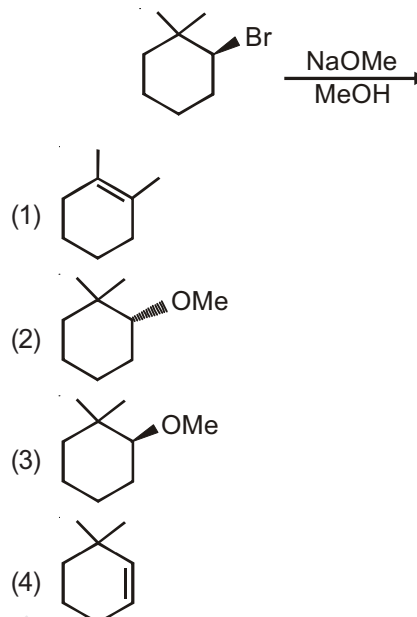
Answer (4)

Sol.



So option (4) is correct.

75. The major product of the following reaction is

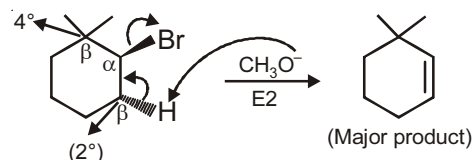


Answer (4)

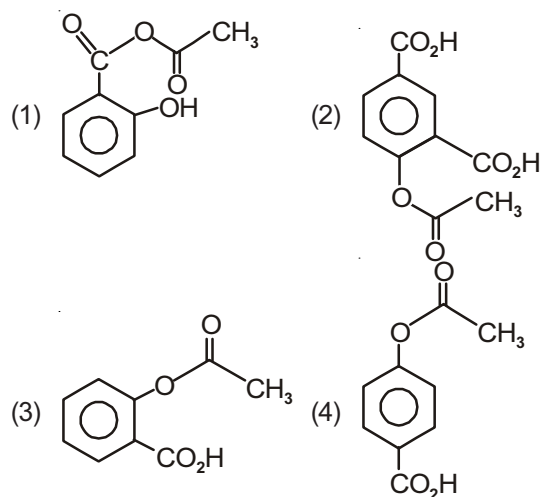
Sol. CH_3O^- is a strong base and strong nucleophile, so favourable condition is $\text{S}_{\text{N}}2/\text{E}2$.

Given alkyl halide is 2° and β C's are 4° and 2°, so sufficiently hindered, therefore, E2 dominates over S_N2.

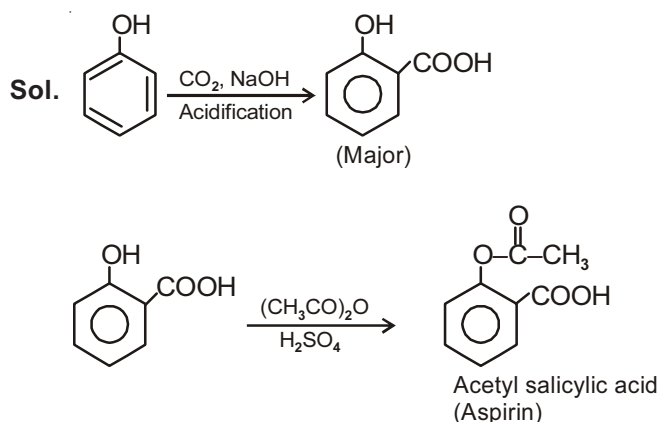
Also, polarity of CH_3OH (solvent) is not as high as H_2O , so E1 is also dominated by E2.



76. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(\text{CH}_3\text{CO})_2\text{O}$ in the presence of catalytic amount of H_2SO_4 produces



Answer (3)



77. An aqueous solution contains an unknown concentration of Ba^{2+} . When 50 mL of a 1 M solution of Na_2SO_4 is added, BaSO_4 just begins to precipitate. The final volume is 500 mL. The solubility product of BaSO_4 is 1×10^{-10} . What is original concentration of Ba^{2+} ?

- (1) $1.1 \times 10^{-9} \text{ M}$
- (2) $1.0 \times 10^{-10} \text{ M}$
- (3) $5 \times 10^{-9} \text{ M}$
- (4) $2 \times 10^{-9} \text{ M}$

Answer (1)

Sol. Final concentration of $[\text{SO}_4^{2-}] = \frac{[50 \times 1]}{[500]} = 0.1 \text{ M}$

K_{sp} of BaSO_4 ,

$$[\text{Ba}^{2+}][\text{SO}_4^{2-}] = 1 \times 10^{-10}$$

$$[\text{Ba}^{2+}][0.1] = \frac{10^{-10}}{0.1} = 10^{-9} \text{ M}$$

Concentration of Ba^{2+} in final solution = 10^{-9} M

Concentration of Ba^{2+} in the original solution.

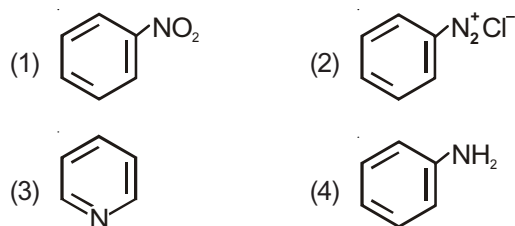
$$M_1 V_1 = M_2 V_2$$

$$M_1 (500 - 50) = 10^{-9} (500)$$

$$M_1 = 1.11 \times 10^{-9} \text{ M}$$

So, option (1) is correct.

78. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?



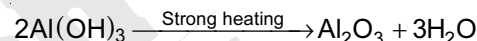
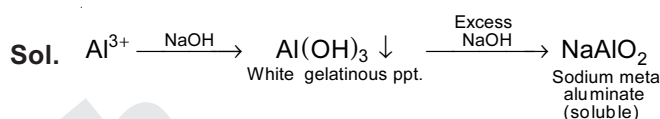
Answer (4)

Sol. Kjeldahl method is not applicable for compounds containing nitrogen in nitro, and azo groups and nitrogen in ring, as N of these compounds does not change to ammonium sulphate under these conditions. Hence only aniline can be used for estimation of nitrogen by Kjeldahl's method.

79. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is

- (1) Al
- (2) Fe
- (3) Zn
- (4) Ca

Answer (1)



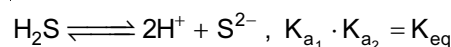
Al_2O_3 is used in column chromatography.

80. An aqueous solution contains 0.10 M H_2S and 0.20 M HCl. If the equilibrium constant for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is

- (1) 6×10^{-21}
- (2) 5×10^{-19}
- (3) 5×10^{-8}
- (4) 3×10^{-20}

Answer (4)

Sol. In presence of external H^+ ,



$$\therefore \frac{[\text{H}^+]^2 [\text{S}^{2-}]}{[\text{H}_2\text{S}]} = 1 \times 10^{-7} \times 1.2 \times 10^{-13}$$

$$\frac{[0.2]^2 [\text{S}^{2-}]}{[0.1]} = 1.2 \times 10^{-20}$$

$$[\text{S}^{2-}] = 3 \times 10^{-20}$$

81. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[\text{3Ca}_3(\text{PO}_4)_2 \cdot \text{Ca(OH)}_2]$ to

- (1) $[\text{3Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$
- (2) $[\text{3}\{\text{Ca(OH)}_2\} \cdot \text{CaF}_2]$
- (3) $[\text{CaF}_2]$
- (4) $[\text{3(CaF}_2) \cdot \text{Ca(OH)}_2]$

Answer (1)

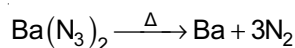
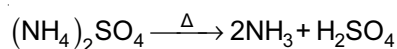
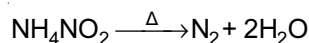
Sol. F^- ions make the teeth enamel harder by converting $[\text{3Ca}_3(\text{PO}_4)_2 \cdot \text{Ca(OH)}_2]$ to $[\text{3Ca}_3(\text{PO}_4)_2 \cdot \text{CaF}_2]$
 Hydroxyapatite Fluorapatite

82. The compound that does not produce nitrogen gas by the thermal decomposition is

- (1) NH_4NO_2 (2) $(\text{NH}_4)_2\text{SO}_4$
 (3) $\text{Ba}(\text{N}_3)_2$ (4) $(\text{NH}_4)_2\text{Cr}_2\text{O}_7$

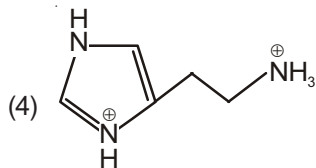
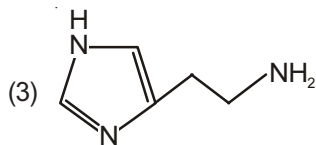
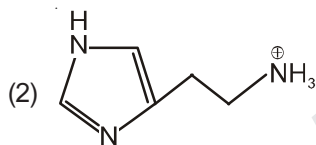
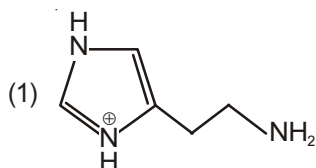
Answer (2)

Sol. $(\text{NH}_4)_2\text{Cr}_2\text{O}_7 \xrightarrow{\Delta} \text{N}_2 + 4\text{H}_2\text{O} + \text{Cr}_2\text{O}_3$

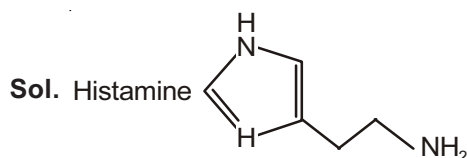


Among all the given compounds, only $(\text{NH}_4)_2\text{SO}_4$ do not form dinitrogen on heating, it produces ammonia gas.

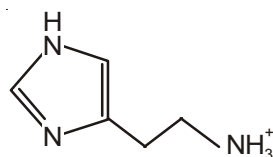
83. The predominant form of histamine present in human blood is (pK_a , Histidine = 6.0)



Answer (2)



At pH (7.4) major form of histamine is protonated at primary amine.



84. The oxidation states of

Cr in $[\text{Cr}(\text{H}_2\text{O})_6]\text{Cl}_3$, $[\text{Cr}(\text{C}_6\text{H}_6)_2]$, and

$\text{K}_2[\text{Cr}(\text{CN})_2(\text{O})_2(\text{O}_2)(\text{NH}_3)]$ respectively are

- (1) +3, 0 and +6 (2) +3, 0 and +4
 (3) +3, +4 and +6 (4) +3, +2 and +4

Answer (1)

Sol. $[\text{Cr}(\text{H}_2\text{O})_6]\text{Cl}_3 \Rightarrow x + 0 \times 6 - 1 \times 3 = 0$

$$\therefore x = +3$$

$[\text{Cr}(\text{C}_6\text{H}_6)_2] \Rightarrow x + 2 \times 0 = 0$

$$x = 0$$

$\text{K}_2[\text{Cr}(\text{CN})_2(\text{O})_2(\text{O}_2)(\text{NH}_3)]$

$$\Rightarrow 1 \times 2 + x - 1 \times 2 - 2 \times 2 - 2 \times 1 = 0$$

$$\Rightarrow x - 6 = 0$$

$$x = +6$$

85. Which type of 'defect' has the presence of cations in the interstitial sites?

- (1) Frenkel defect
 (2) Metal deficiency defect
 (3) Schottky defect
 (4) Vacancy defect

Answer (1)

Sol. In Frenkel defect, cation is dislocated from its normal lattice site to an interstitial site.

86. The combustion of benzene (l) gives $\text{CO}_2(\text{g})$ and $\text{H}_2\text{O}(\text{l})$. Given that heat of combustion of benzene at constant volume is $-3263.9 \text{ kJ mol}^{-1}$ at 25°C ; heat of combustion (in kJ mol^{-1}) of benzene at constant pressure will be

$$(R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1})$$

- (1) 3260 (2) -3267.6
 (3) 4152.6 (4) -452.46

Answer (2)

Sol. $\text{C}_6\text{H}_6(\text{l}) + \frac{15}{2}\text{O}_2(\text{g}) \longrightarrow 6\text{CO}_2(\text{g}) + 3\text{H}_2\text{O}(\text{l})$

$$\Delta n_g = 6 - \frac{15}{2} = -\frac{3}{2}$$

$$\Delta H = \Delta U + \Delta n_g RT$$

$$= -3263.9 + \left(-\frac{3}{2}\right) \times 8.314 \times 298 \times 10^{-3}$$

$$= -3263.9 + (-3.71)$$

$$= -3267.6 \text{ kJ mol}^{-1}$$

87. Which of the following are Lewis acids?

- (1) PH_3 and SiCl_4
- (2) BCl_3 and AlCl_3
- (3) PH_3 and BCl_3
- (4) AlCl_3 and SiCl_4

Answer (2)*

Sol. BCl_3 – electron deficient, incomplete octet

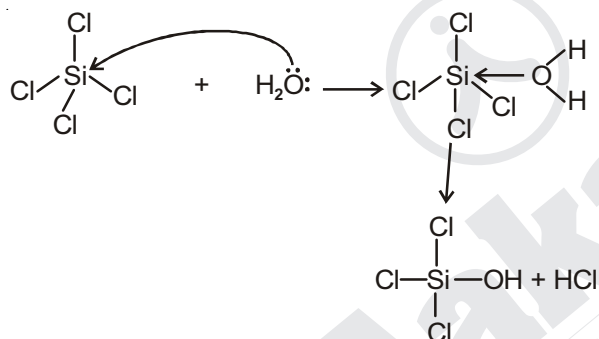
AlCl_3 – electron deficient, incomplete octet

Ans-(2) BCl_3 and AlCl_3

SiCl_4 can accept lone pair of electron in d -orbital of silicon hence it can act as Lewis acid.

* Although the most suitable answer is (2). However, both option (2) & (4) can be considered as correct answers.

e.g. hydrolysis of SiCl_4



Hence option (4), AlCl_3 and SiCl_4 is also correct.

88. Which of the following compounds contain(s) no covalent bond(s)?

KCl , PH_3 , O_2 , B_2H_6 , H_2SO_4

- (1) KCl
- (2) KCl , B_2H_6
- (3) KCl , B_2H_6 , PH_3
- (4) KCl , H_2SO_4

Answer (1)

Sol. KCl – Ionic bond between K^+ and Cl^-

PH_3 – Covalent bond between P and H

O_2 – Covalent bond between O atoms

B_2H_6 – Covalent bond between B and H atoms

H_2SO_4 – Covalent bond between S and O and also between O and H.

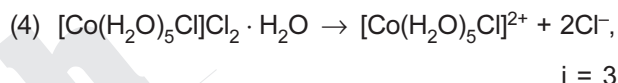
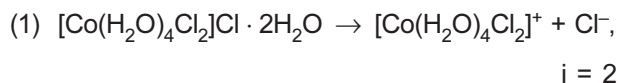
$\therefore$ Compound having no covalent bonds is KCl only.

89. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?

- (1) $[\text{Co}(\text{H}_2\text{O})_4\text{Cl}_2]\text{Cl} \cdot 2\text{H}_2\text{O}$
- (2) $[\text{Co}(\text{H}_2\text{O})_3\text{Cl}_3] \cdot 3\text{H}_2\text{O}$
- (3) $[\text{Co}(\text{H}_2\text{O})_6]\text{Cl}_3$
- (4) $[\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2 \cdot \text{H}_2\text{O}$

Answer (2)

Sol. The solution which shows maximum freezing point must have minimum number of solute particles.



So, solution of 1 molal $[\text{Co}(\text{H}_2\text{O})_3\text{Cl}_3] \cdot 3\text{H}_2\text{O}$ will have minimum number of particles in aqueous state.

Hence, option (2) is correct.

90. According to molecular orbital theory, which of the following will not be a viable molecule?

- (1) H_2^-
- (2) H_2^{2-}
- (3) He_2^{2+}
- (4) He_2^+

Answer (2)

Sol.

	Electronic configuration	Bond order
He_2^+	$\sigma_{1s}^2 \sigma_{1s}^{*1}$	$\frac{2-1}{2} = 0.5$
H_2^-	$\sigma_{1s}^2 \sigma_{1s}^{*1}$	$\frac{2-1}{2} = 0.5$
H_2^{2-}	$\sigma_{1s}^2 \sigma_{1s}^{*2}$	$\frac{2-2}{2} = 0$
He_2^{2+}	σ_{1s}^2	$\frac{2-0}{2} = 1$

Molecule having zero bond order will not be a viable molecule.